The interaction between the central bank and government in tail risk scenarios

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March 2013

Abstract

This paper presents an original approach of the strategic interaction between governments and the central bank with regard to unconventional monetary policy. Crisis measures to remove tail risk are analysed with an adjusted Merton model in a game theoretical set-up. It shows that the participation constraint for interventions by the central bank and the governments is less binding if the risk of contagion is high. The strategic interaction between governments and the central bank also influences the effectiveness of the interventions. A joint effort of both the governments and central bank leads to a better outcome. To prevent a bad equilibrium, a sizable commitment by both players is required. The outcomes help to understand the dynamics behind the Outright Monetary Transactions (OMTs) of the Eurosystem.

Keywords: Financial crisis, Monetary policy, Central banks, Policy coordination
JEL Classification: E42, E52, E61, G01, G18

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The views expressed are those of the authors and do not necessarily reflect official positions of DNB. The authors would like to thank Peter van Els, Jakob de Haan and participants of the DNB lunch seminar for valuable comments.
1. Introduction

The rise of sovereign bond spreads of several EMU countries since 2010 indicates that stress in sovereign bond markets has increased. An important reason for this stress is that the tail risk of a sovereign default has increased. In a currency union this goes in tandem with risks of contagion due to the financial integration of member states. According to the IMF (2012), the default of an EMU member state is a tail risk that could lead to contagion of other states and bank runs, threatening the EMU as a whole. The severe distortions in government bond markets have indeed raised fears of the reversibility of the euro, which was the prime reason for the ECB to enact Outright Monetary Transactions (OMTs) as a backstop to avoid destructive scenarios (ECB, 2012). Market participants have published scenarios of such events (e.g. Euro Intelligence, 2009, ING, 2011 and 2012, Buiter, 2011). These scenarios describe possible ways in which a sovereign default within the currency union could unfold, including the potential costs. The estimations are based on crude assumptions of determining factors, such as the conditions for debt restructuring. These assumptions and the probability attached to the sovereign risk scenarios are highly judgmental. In some analyses the policy of central banks and governments play a role, as ex ante assumptions driving the scenarios. However, the interactions between policy measures and the evolution of scenarios are not modelled.

The academic literature provides no objective framework for the dynamics of sovereign risk scenarios in a currency union that includes the influence of policy. Eichengreen (2007) describes the dynamics in scenarios based on economic incentives which countries have to leave the EMU, such as the possibility to restore their competitiveness by currency depreciation. However, he does not discuss that the probability and costs of scenarios are endogenous to the reaction of policymakers. Friedman (2002) took into account an extreme scenario already at the start of EMU. Krugman (2010) points to the importance of decisive policy reactions, but does not explain how this could mitigate tail risks. In a way this is done by Kapp (2012), who shows in a general equilibrium model how the European Stability Mechanism (ESM) can insure vulnerable EMU countries from sudden stops in capital inflows. In his model the central bank plays no role. The interaction between central banks and governments in normal circumstances is discussed in literature by Alesina (1988) and, more recently, in the literature on fiscal dominance and the interconnections between government finances, government debts and monetary policy (see for an overview BIS, 2012). Dixit and Lambertini (2001) model the non-cooperative interaction between monetary and fiscal authorities in a monetary union. They find that giving a leadership role to either authority leads to a more desirable outcome.

The added value of our paper is that we model the probability and costs of sovereign risk scenarios, taking into account the influence of unconventional crisis measures taken by the central bank and by the governments. In our model, we explicitly include the interaction between policymakers that is typical in the context of the current European debt crisis. This fills a gap in the literature, which usually focusses on interactions between the central bank and the government in the
context of conventional monetary policy. Our model approach focusses on two research questions. First we analyse the participation constraint for central bank interventions that aim to prevent tail risk scenarios that arise by sovereign risk. Such scenarios entail economic and financial costs, including potential contagion effects across the currency union, and these costs determine a participation constraint for central bank interventions. The interventions reduce the likelihood of a tail event, but shifts risks to the balance sheet of the central bank. Before deciding to intervene, the central bank employs an adjusted version of the Merton model to compare the costs of a sovereign default at $t = 0$ with the related exposure risk. The central bank exposures are weighted by the likelihood that its interventions are not effective at the end of the day. A crucial feature in the model is that this probability is endogenous to the central bank measures. So we take into account that the effort of the central bank influences the probability of tail risk scenarios.

The second research question concerns how support measures taken by the governments interact with the interventions by the central bank to influence the probability of sovereign risk scenarios. This sheds light on the interaction of governments and the Eurosystem in the European debt crisis, where governments embark on support programs in vulnerable EMU countries, financed by the European Stability Mechanism (ESM) and the ECB makes its unconventional monetary policy in the context of Outright Monetary Transactions (OMTs) conditional on activation of the ESM. To capture the interaction between the central bank and the government we extend the model to a game theoretical framework. Herewith we analyse the probability and costs of sovereign risk, including the different interests and incentives of the central bank and the governments. The model is a stylized analysis that focusses on strategic interactions between both players. It does not explicitly address other related issues, for instance with regard to monetary financing of governments.

The model shows that the probability of a sovereign risk scenario is reduced by a continuation of crisis measures. Their effectiveness depends on the interaction between the central bank and government. The participation constraint that both the governments and the central bank are facing, is more binding in case of a small country default than in case of a large country default. If we take into account that a small country default engenders contagion to other countries, the central bank has additional room to intervene. Including the government in the model illustrates the interaction between both players. If either the government or central bank contributes less than a certain threshold it is optimal for the other player to contribute nothing. This does not solve the crisis. It is optimal for both players to combine their efforts. In the optimum the likelihood of a sovereign risk scenario declines and expected loss on the exposures of both players is limited. In the Stackelberg equilibrium with the governments as leader (which mimics the ECB OMT program) the marginal effect of the central bank interventions rises, by which the central bank has an incentive to enlarge its effort as well. This leads to a more desirable outcome, as is the case in Dixit and Lambertini (2001). In a cooperative solution, where the governments and central bank form a coalition to maximize the joint utility of both players, their utility is very close to the one obtained in the Stackelberg equilibrium.
The rest of the paper is organised as follows. In Section 2 we describe the structure and assumptions of the model. Section 3 presents the outcomes of the model for different scenarios in which the central bank is the only party that influences the tail risk probability. In Section 4 the model is extended with the government. It shows that the interaction between the central bank and the government determines the outcomes of extreme scenarios. Section 5 concludes.

2. Framework

2.1 Assumptions

Interventions by governments and/or by the central bank(s) may reduce the likelihood (P) that a tail event occurs. If the time bought by the interventions is well spent and troubled countries and banks can grow out of their debt problems, they may regain access to the capital market. However, if that does not happen, the exposures of the supporting governments \((G_n)\) and/or central bank \((CB_n)\) rise further, as well as the probability of a tail risk. In the model, the public costs of sovereign risk scenarios at \(t = 0\) are compared to the expected loss of the central bank related to its crisis interventions. Total direct costs \(L_0\) at \(t = 0\) consist of three components. The public costs of a tail risk scenario at \(t = 0\) are \(S_0\). These are the costs to society, which instantaneously arise due to disruptions in the economy and in the financial system after a sovereign default. Estimations of those costs are highly uncertain. According to the IMF (2012) a sovereign default of an EMU country poses a risk to the currency union. Sovereign risk may give rise to negative confidence effects that lead to destabilising capital flows, impaired monetary transmission and threaten the financial position of other countries in the currency union. This would justify crisis measures taken by the central bank and governments. Some market participants estimate that default of a small EMU country (and its exit from EMU) would lead to a loss of 5% of (euro area) GDP while the costs of multiple sovereign defaults which threaten the currency union could amount to 12% GDP. A default at \(t = 0\) would also lead to direct losses to the central bank \((CB_0)\) and to the government \((G_0)\) on their outstanding exposures to the country that defaults. \(G\) refers to the total exposure of supporting governments (extended through the EFSF/ESM) on countries at risk of default. Total direct costs at \(t = 0\) equal \(L_0 = S_0 + CB_0 + G_0\).

If the interventions by the government and/or central bank turn out to be not effective and a tail risk materialises at \(t = n\), the economic costs rise to \(S_n\). In this case, the costs to society are larger \((S_n > S_0)\) since in years \(t = 0..n\) the costs accumulate due to output losses related to failed economic adjustment programs, ongoing uncertainty and falling asset prices. The costs to the supporting government \(G\) increase (with \(G_n > G_0\)) due to losses on EFSF/ESM support extended during \(t = 0..n\). These can either be losses on disbursements under a macroeconomic adjustment program, a contingent credit line or primary market purchases. The expected cost of a default increases with higher \(G\), since
The expected loss of the central bank (CB) also increases (with $CB_n > CB_0$) due to higher exposures related to asset purchases, refinancing operations and emergency liquidity assistance to banks. During $t = 0..n$ the underlying risks of the central bank could further rise due to a shift of exposures to weak countries or banks and to the pledging of lower quality collateral in refinancing operations. Total costs at $t = n$ equal $L_n = S_n + CB_n + G_n$.

While in this set-up the variables $S$, $CB$ and $G$ independently determine $L$, in the next section we show that the expected loss $E(L)$, and thereby the expected values of $S$, $CB$ and $G$, are endogenously determined by the probability of a tail event, which itself is a function of $CB$ and $G$. Hence, in the model the economic costs are influenced by the policy interventions. It also reflects a shift of private costs ($S$) to potential costs for the government ($G$) and central bank ($CB$) and presents an inter-temporal distribution issue since it is likely that the authorities would tax the private sector after the crisis to restore their financial position again.

2.2 Model

The total expected loss is determined by the probability $P$ that the tail risk occurs, with $P$ being a reflection of sovereign default risk. $P$ is modelled by applying the Contingent Claims Analysis of Merton (1974). Herein a balance sheet of an entity is described in terms of the option theory. Equity is perceived as a call option on the underlying assets ($A$). When the market value of $A$ exceeds the nominal debt ($DB$) the option is in the money. A default occurs if the equity value declines and $A$ becomes lower than $DB$. The default risk is summarised in one measure, the distance to distress ($d$),

$$d = \frac{\ln\left(\frac{A}{DB}\right) + \left(r + \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}$$

$P$ indicates how many standard deviations the market value of assets ($A$) is away from the default point (nominal debt level, $DB$), with $T$ the time horizon (usually fixed at 1 year) and $r$ the risk free 12 month interest rate. The default risk increases if the value of assets declines, the debt increases or if the volatility of the asset value ($\sigma_A$) rises. The distance to distress can be expressed as the probability of default ($P$),

$$P = N(-d)$$

whereby the Merton model is based on a standard normal distribution $N(.)$. The Contingent Claims Analysis is usually applied to corporations. A well-known limitation of the model is the dependency
on market prices, which may give misleading signals about the underlying risks. Other drawbacks of the standard Merton model are the use of a short risk horizon of one year and the assumption that default occurs only at the maturity of the debt. Gapen et al. (2008) apply the model to sovereigns. In those applications the asset value is unknown and should be derived from other balance sheet items which can be quantified by approximation, such as own funds (proxied by the domestic currency liabilities of a country) and debt (proxied by foreign currency liabilities). Although these are crude proxies of the model variables, it avoids the drawback of using market prices.

We use the application by Gapen et al. to model the probability of sovereign default where the governments and the central banks take measures to prevent an extreme outcome. To this end we extend the framework by modelling the dependency between the probability of default \( P \) and the efforts of the central bank and the government. On the one hand the interventions by the central bank (\( CB \)) influence tail risk. The central bank provides liquidity support to underpin the functioning of banks and markets (the crisis measures of the central bank are aimed at supporting the monetary transmission through banks and the bond market). In terms of the Merton model this liquidity support can reduce the volatility of the market values of assets (\( \sigma_A \)). On the other hand, \( P \) determines the expected loss of the central bank, since a sovereign default affects the value of central bank exposures. \( P \) is also dependent on the support provided by governments (\( G \)) to vulnerable countries. Their support can contribute to confidence and mitigate sovereign default risk. The crisis measures of governments are primarily aimed at improving the debt sustainability of countries. This is realised by financial support and consolidation and reform programs. In the model we assume that this support influences the nominal debt level, \( DB \). The adjusted model then becomes,

\[
d = \frac{\ln \left( \frac{A}{f(G) DB} \right) + \left( r + \frac{1}{2} (f(CB) \sigma_A)^2 \right) T}{f(CB) \sigma_A \sqrt{T}}
\]

in which the intervention functions \( f(CB) \) and \( f(G) \) determine the effect of policy interventions on tail risk (\( P \)).

\[
f(G) = \alpha + e^{\beta G^2 + \gamma G}
\]

\[
f(CB) = \alpha + e^{\beta CB^2 + \gamma CB}
\]

With \( \beta > 0 \) and \( \gamma < 0 \). Equation 4 implies that an extension of crisis measures (\( G \) or \( CB \)) first reduces the probability of default risk, but after some point goes in tandem with rising tail risk probabilities (Figure 1). In our simulations we use the parameter values that are in line with the values used by Gapen et al. We use for the volatility of assets \( \sigma_A = 0.5 \), for the risk-free rate \( r = 4\% \) and \( T=1 \). With regard to \( CB \) this assumes that from a certain point the credibility of the central bank diminishes (for
instance due to reduced possibilities to sterilise the over liquidity or due to an increasing risk of a negative equity position), which raises tail risk. A similar convex relationship is assumed between government support ($G$) and tail risk. Increasing support affects the financial capacity of the supporting government and may lead to a downgrading of its credit rating. If the central bank and the government both intervene, the mitigating effect of the crisis measures on tail risk is reinforced. That situation is explored in Section 4.

The model has a time dimension and a participation constraint. The participation constraint states that the government and/or central bank will only intervene if the expected loss on their interventions during $t = 0..n$ does not exceed the costs related to realisation of an extreme scenario at $t = 0$. This restriction is presented in equation 5.

$$ E_0(L_n) < L_0 $$  \hspace{1cm} (5)

with $E_0(L_n) = P(S_n + CB_n + G_n)(1 + r)^{-n}$ which expresses that the expected loss of the central bank is dependent on its exposure ($CB_n$). This exposure influences tail risk ($P$) through the relationship in Equation 4, by which $P$ is endogenous to the central bank interventions, as well as to government interventions. The expected loss at $t = 0$ $E_0(L_n)$ is equal to the expected present value of the total loss accumulated over $t = 0..n$.

3. Outcomes

3.1 Extreme scenarios

The model is used to analyse the question whether the expected costs of extended interventions outweigh the total costs of a sovereign risk scenario at $t = 0$. These costs could materialise if the central bank stops to intervene and/or government support is withdrawn. The economic and financial
costs of default are approximated by the loss on government and central bank exposures \((G + CB)\) and the economic costs \((S)\) that could materialise in an extreme scenario. In this stage we only take into account the effects of central bank interventions and abstract from government support \((G_0 = G_a = 0)\).

The first scenario simulates a small country in risk of default and the effect of crisis measures taken by the central bank. We analyse how much room the central bank has to mitigate the tail risk by extending its exposure. This room is limited by the participation constraint in equation 5. Figure 2 shows that at a higher exposure (movement along the loss curve to the right) the expected loss for the central bank and society due to continued crisis interventions during \(t = 0..n\) eventually exceeds the total costs of a tail event occurring at \(t = 0\) \((L_0, \text{ the limit presented by the horizontal line})\). This limit, the participation constraint, depends on the probability of default \((P)\). A high initial probability (e.g. 30%) constrains the central bank at an earlier stage. If the initial probability is half as large (15 instead of 30%), the central bank could extend its exposures under the limit much more.

The second scenario simulates a large country default, taking into account the related potential economic and exposures losses at \(t = 0\). If the initial probability of default would equal 5%, the central bank could extend its exposure indefinitely during \(t = 0..n\) before the expected loss would hit the constraint, as depicted by the horizontal line in Figure 3. This conclusion is based on the assumed low initial probability of sovereign default \((P)\). If this probability rises, the optimal effort of the central bank is bounded at an earlier stage, as shown by the loss curves crossing \(L_0\) at initial default probabilities of 15 and 30% in Figure 3.

### 3.2 Contagion effects

The participation constraint for central bank interventions to prevent default of a small country is less binding when we take into account that the shock may put pressure on the currency union through contagion effects. Research shows that the default probability of an EMU country is two or three times higher if another small EMU country defaults (Zhang et al, 2011). We define the conditional expected loss of multiple sovereign defaults \((L_{10})\) given a small country default at \(t = 0\) as \(E(L_{10}|P_{S0} = 1)\). The cost of contagion increases the costs of default of a small country and thereby
relaxes the participation constraint for central bank interventions. This is reflected by adding the conditional costs of a multiple defaults ($L_{L0} = S_{L0} + CB_{L0} + G_{L0}$) at $t = 0$ to the constraint in equation (6),

$$E_0(L_{S0}) < L_{S0} + E(L_{t=0} | P_{S0} = 1)$$  \quad (6)$$

From equation (6) it follows that when contagion effects are taken into account, the central bank can extend its exposures more before the constraint is hit. Figure 4 shows that the constraint, i.e. the horizontal line, shifts upwards. So when contagion risk is taken into account, central bank exposures can be extended further to prevent possible chain reactions within the currency union.

4. Interaction with the government

4.1 Framework

Tail risks are not only influenced by central bank measures, but also by governments supporting vulnerable countries. In the model, government support ($G_n$) influences sovereign default risk ($P$) through its effect on the nominal debt level ($DB$) according to equations 3 and 4. The interventions by the central bank and by the governments complement each other. If the effort of the government is combined with the interventions by the central bank, the marginal (downward) effect on the probability of default increases. This effect is included by extending the intervention functions $f(CB)$ and $f(G)$ presented in equation 4 with interaction term $CB * G$,

$$f(CB) = \alpha + e^{\beta CB^2 + \gamma CB + \eta CB^* G}$$ \quad (7)$$

$$f(G) = \alpha + e^{\beta G^2 + \gamma G + \eta CB^* G}$$
With $\beta > 0$, $\gamma < 0$ and $\eta < 0$. The interaction term expresses that policy measures aimed at mitigating sovereign risk are most effective at combined efforts of the central bank and government. Central banks can address liquidity problems and buy time, while governments should address structural economic and financial problems that drive sovereign risk and block monetary transmission. For instance through reform programs that improve debt sustainability of countries and support the functioning of markets. Since both the central bank and government influence sovereign risk, the interaction between both players is important. In the current crisis, the ECB has encouraged governments to take action and deal with structural problems in the banking sector and government budgets. An example are the Outright Monetary Transactions (OMTs) of the ECB, in which central bank interventions in bond markets are conditional on government actions through the ESM. The conditionality should limit moral hazard risk on the side of governments who naturally prefer that the central bank intervenes since this minimises their budget outlays.

The strategic interaction between the central bank and government and their incentives can be described by a game theoretical model. Herein, the central bank and the government play a non-cooperative game in which they determine their optimal effort, minimising their own expected loss ($L^{CB}$ and $L^{G}$) in a sovereign risk scenario subject to the participation constraint,

\[
E_0 (L^G) = (S_n + wG_n + CB_n)(1 + r)^{-n} P < L_0
\]

\[
E_0 (L^{CB}) = (S_n + G_n + wCB_n)(1 + r)^{-n} P < L_0
\]  

In this game theoretical model we analyse both the Nash equilibrium and the Stackelberg equilibrium. In the latter we assume that the government is the leader. The leader in the game is committed to its intervention and cannot react to the follower’s action. This fits with the common view that the governments are less flexible in dealing with the crisis than the central bank. The leadership of the government is also in line with the philosophy of the OMT program of the ECB which is conditional on initiatives by governments. The central bank in its role as follower determines its effort by taking the effort of the government as given. The government and central bank minimize almost identical loss functions, equal to the present value of the contributions of both players, multiplied by the probability of default. The only difference between the loss function of both players is the weight $w \geq 1$ that indicates how much each player values the loss of its own contribution relative to the loss of the other player. This introduces an incentive for each player to shift the burden to the other player.

The Stackelberg game is solved by backward induction, taking an effort of the follower ($CB$) which minimises the loss, given the effort of the leader ($G$). The FOC for the central bank, in both the Nash and the Stackelberg game, is
\[
\frac{dL^{CB}}{dCB} = \frac{dP(CB,G)}{dCB} (wCB + G + S) + Pw = 0
\]  

(10)

The leader (G) determines his own effort to minimise its loss; the model assumes that the leader is committed to that and cannot change its effort. In his reaction function the government as leader takes into account the expected effort of the follower (which is dependent on the effort of the leader),

\[
L^G = P(G, CB|G)) (S + CB + wG)
\]  

(11)

The FOC of the government in the Stackelberg game is,

\[
\frac{dL^G}{dG} = \frac{dP(CB|G), G)}{dG} (wG + CB + S) + P\left(\frac{dCB|G}{dG} + w\right) = 0
\]  

(12)

And for the Nash game:

\[
\frac{dL^G}{dG} = \frac{dP(CB,G)}{dG} (wG + CB + S) + Pw = 0
\]  

(13)

The first term on the right hand side of equation (12) states that the government as Stackelberg leader influences the probability of default through two channels. Its effort \(G\) affects the probability of default directly and indirectly through its influence on the effort of the central bank. The effect of government intervention is thus larger than in the Nash game and this also gives the central bank an incentive to extend its efforts. The last term of equation (12) reflects that the Stackelberg leader, besides its influence on the probability of a tail event also has more influence on the loss given default. The government chooses its own contribution (weighted by \(w\) in the government’s loss function) and thereby affects the contribution by the central bank. Since the central bank’s reaction function is upward sloping, a higher contribution by the government increases loss given default by more than \(w\). This latter effect is an incentive for the government to lower its effort.

4.2 Optimal effort

The FOCs are solved by using the following parameter values in equation 3 (distance to distress) and equation 7 (i.e. intervention functions \(f(CB)\) and \(f(G)\)): \(P = 5\%\) (initial default risk) and \(\alpha = 0.2, \beta = 6.25, \gamma_{CB} = -2.5, \gamma_G = -2, \eta = -4, \sigma_A = 0.5\) and \(w = 3\). Based on these values the probability function \(P = \mathcal{N}(-d)\) is presented in Figure 5.
The x-axis (right to left) shows the effort of the central bank and the y-axis (back to front) the effort of the government. The vertical z-axis reflects probability $P$. With a positive effort by the central bank (and a certain government exposure) the probability first declines, but after a certain exposure it increases again. This is similar to the pattern in Figure 1. However, if both the central bank and government provide more support (movement along x and y-axes) the increase of the default probability is limited, while at a large effort by both players it remains stable at a low level. This follows from the strengthened effect on the distance to distress in equation 3 and the interaction terms in equation 7 when both the central bank and the government intervene. Their joint effort leads to a better outcome.

The total expected loss shows up as in Figure 6.
The x-axis (right to left) shows the effort of the government and the y-axis (back to front) the effort of the central bank. The horizontal plane in the graph is the participation constraint that is determined by a strategy of non-intervention (equal to the costs of multiple sovereign defaults at $t = 0$). This limit ($L_0$) equals the economic costs of multiple defaults, plus the losses of the central bank and the government in that situation ($L_0 = S_0 + G_0 + CB_0$). The optimal effort of the central bank and government must remain under this limit. The social cost function in Figure 6 is a function of the unweighted effort of the central bank and the government. The expected social cost is determined by multiplying the combined exposures by the initial probability of an extreme scenario. It shows a similar pattern as Figure 5: if both the central bank and government provide more support (movement along x and y-axes in Figure 6) the expected loss of their interventions remains below the participation constraint.

**Figure 7. Reaction functions central bank and government**

Figure 7 illustrates the interaction between the central bank ($CB$) and government ($G$). The vertical axis represents the central bank’s contribution ($CB$). If either the central bank or the government contributes less than $E^*$ the optimal reaction by the other party is to contribute nothing. In that case, the initial contribution is too low and the probability of an extreme scenario remains too high so that any positive contribution by the other player will result in an expected loss. In this case, the players end up in the bad Nash-equilibrium at the origin where both players do nothing to prevent an extreme outcome. As long as the government contributes more than $E^*$, the central bank will react with a
positive contribution, according to the central bank’s reaction function (the blue line). The central bank’s reaction will be incorporated by the government, according to its reaction function (the red line). Eventually, both players will settle down in the good Nash-equilibrium N. Alternatively, in the Stackelberg game where the government is the leader, the government chooses point S on the central bank’s reaction function so as to minimize its loss-function $L^G$.

The optimal effort of the central bank ($CB$) depends on the effort of the government ($G$). In the good Nash equilibrium ($N$) both players choose their exposures so that a unilateral deviation of the strategy does not result in a better outcome. With the chosen parameter values, the probability of an extreme scenario then is 4.1%. In the Stackelberg equilibrium ($S$) with the government as leader, the government determines its effort, taking into account the reaction of the central bank. In this case the probability of sovereign risk scenario is reduced to 3.4%.

By determining his effort first in the Stackelberg game, the leader can reduce the probability of an extreme scenario as well as the expected loss. Since the government commits itself as leader in the Stackelberg equilibrium to a higher effort than in the Nash equilibrium, it raises the marginal effect of the effort by the central bank. By this, the central bank has an incentive to enlarge its effort as well. The total exposures of central bank and government together are much higher in the Stackelberg equilibrium than in the Nash equilibrium, by which the likelihood of a successful intervention increases. Hence despite the higher efforts, the expected loss for both players is lower in the Stackelberg equilibrium.

In a cooperative setting, the governments and the central bank form a coalition that maximizes the joint utility of both players. The challenge is to ensure that the joint payoff is split in a way that makes both players better off than in the non-cooperative setting. In the non-cooperative game, both players have a similar loss function which differs only in the larger weights that is attached to the player’s own contribution. In the joint loss function for the coalition, the contributions of both players have an identical weight. In the cooperative solution, the central bank contributes 49.6% and the governments contribute 39.2%. This solution makes both players better off than in the Nash-equilibrium (6.3% vs 6.6% for the central bank loss and 5.6% vs 5.8% for the governments). The probability of the sovereign risk scenario is 3.3% in the cooperative outcome. The central bank is slightly better off in the Stackelberg equilibrium.

4.3 Sensitivity analysis

As robustness test the outcomes of the Nash equilibrium and the Stackelberg game are simulated with alternative parameter values in the probability function in equation 7. The parameter values used in the baseline scenario have been calibrated so that they yield plausible outcomes. Table 1 shows that the conclusions in the previous section are robust, i.e. a joint effort of the central bank and government reduce the probability of a tail event significantly and the Stackelberg game results in a lower loss than the Nash game. The first row in Table 1 shows the base scenario. The second row shows that a lower
value of β leads to a higher effort, since the effect of diminishing credibility due to a higher effort becomes lower. A higher marginal effect (more negative γ) of the effort of both players also results in a higher effort. The same holds if the interaction effect η is increased. A lower weight w on own exposure leads to a higher effort in the Nash equilibrium. This relates to the fact that both players are more inclined to raise their efforts if the marginal costs of their own effort are lower. In the Stackelberg game there is also the effect that the leader knows that the follower will raise its effort when the leaders’ effort stays the same. The result of these two opposite effects is that the leader (G) reduces its effort slightly and the follower (CB) raises it somewhat more. With alternative values of α, β, γ and η the optimal effort of both players thus turns out to be higher, although the probability of an extreme scenario remains below 4%. As a result the loss also remains low.

### Table 1, Sensitivity analysis

<table>
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<th>CB Nash</th>
<th>G Nash</th>
<th>Pe Nash</th>
<th>CB Stack</th>
<th>G Stack</th>
<th>Pe Stack</th>
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<td>2.2%</td>
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<td>η=-4.25</td>
<td>52.1%</td>
<td>42.3%</td>
<td>2.2%</td>
<td>55.2%</td>
<td>45.9%</td>
<td>2.0%</td>
</tr>
<tr>
<td>w=2</td>
<td>45.6%</td>
<td>35.5%</td>
<td>3.7%</td>
<td>48.3%</td>
<td>38.6%</td>
<td>3.4%</td>
</tr>
</tbody>
</table>

Explanation: CB Nash (G Nash) is the effort of the central bank (government) in Nash equilibrium in % of GDP and Pe Nash is probability of tail risk in Nash equilibrium. CB Stack (G Stack) is the effort of the central bank (government) in Stackelberg equilibrium with the government as a leader and Pe Stack is probability of tail risk in Stackelberg equilibrium.

### 5. Conclusions

Based on an adjusted Merton model we present an original approach of the strategic interaction between governments and the central bank with regard to unconventional monetary policy. It shows how the central bank and governments influence the probability and costs of sovereign risk scenarios. A crucial feature in the model is that the probability of such a tail event is endogenous to policy measures. The probability of a tail event is reduced by continuation of crisis interventions and their effectiveness is dependent on the interaction between the central bank and government. If the central bank alone intervenes to solve the crisis, there is a limit beyond which it is not optimal for the central bank to extend its exposure since the expected loss would then exceed the costs of a tail event materialising at t = 0. This limit is reached at an earlier stage in case of a small shock (e.g. small country default), than in case of a large shock, since the economic and financial costs of the latter are worse. If we take into account that the probability of a small country default contaminates other countries – which is particularly a risk in a currency union – the central bank has additional room to extend its crisis interventions.
The interaction between the central bank and the government is analysed in a game theoretical framework. Herein, both players determine their optimal effort, at which they minimise their own loss in a sovereign risk scenario, while there is also a positive interaction effect through which the interventions are increasingly effective at combined efforts of the central bank and government. There is an incentive to shift the burden of the intervention to the other player because the marginal costs of its efforts are higher compared to the marginal costs of the other player.

In the chosen parameter configuration, a joint effort of both players leads to a better outcome. If both the central bank and government provide more support, the expected loss of their interventions remains below the constraint. However, when the initial contribution of one of the players is too low and the probability of an extreme scenario remains too high, so that any positive contribution by the other player will result in an expected loss, the players end up in the bad Nash-equilibrium where both players do nothing to prevent an extreme outcome. This is only solved at a sizable effort by one of the players, which would entice the other player to commit as well. In the Stackelberg game with the government as leader, it can reduce the probability of an extreme scenario as well as the expected loss. By determining its effort first, the government raises the marginal effect of the central bank’s interventions and gives the central bank an incentive to enlarge its effort as well. The total exposures of central bank and government together are much higher in the Stackelberg equilibrium than in the Nash equilibrium, by which the likelihood of a successful intervention increases. Hence despite the higher efforts, the expected loss for both players is lower in the Stackelberg equilibrium. The cooperative solution maximizes the joint utility of both players. In the cooperative solution, both players contribute more than in the Nash and in the Stackelberg equilibrium. Hence, the probability of a sovereign risk scenario is lower. Both players’ utility is very close to the one obtained in the Stackelberg equilibrium, with the government being slightly better and the central bank slightly worse off in the cooperative solution.

The model outcomes can be applied to policy strategies that are being developed to solve the European debt crisis, where governments support vulnerable EMU countries through the ESM and central bank interventions are conditional on activation of the ESM in the context of the ECB’s Outright Monetary Transactions (OMTs). A general result is that a joint effort of the government and central bank leads to a better outcome. A more specific policy conclusion is that a first mover position of the government leads to an outcome that is very close to the optimal cooperative outcome.
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